ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume IX (XIX), 2010, NR3

# ASPECTS REGARDING THE FREE AND FORCED VIBRATIONS OF A PIPELINE SECTION SUBJECTED TO VARIABLE LOADS AND AXIAL FORCE

**POPA Ioan, STANCIU Lavinia Silvia**
Petroleum-Gas University of Ploiesti
ipopa@upg-ploiesti.ro, laviniastanciu@yahoo.com

**Keywords:** pipeline, variable loads, natural frequencies, dynamic response

**Abstract.** The aim of the present paper is to establish the differential equations for a pipeline section subjected to uniform variable loads, perpendicular on its axis, using the Lagrange equations. The normal conditions of temperature and pressure of the external environment are taken into account. The natural frequencies' equation is determined and, also, the natural frequencies` dependence on different dimensional factors. The pipeline section's dynamic transversal displacement response is also determined, pointing out the maximum value of the amplitude in the forced vibration. The influence of the axial force on the natural frequencies is analyzed.

## 1. INTRODUCTION

The pipes through which fluids under pressure flow are submitted to some vibration phenomena, caused by the fluid flow mode when changing directions. In these zones, disturbing forces occur, generating both axial and transverse vibrations. In order to study the free and forced transverse vibrations, we considered a pipeline section through which a fluid having harmonic variations of pressure is flowing. These variations can be transposed in disturbing harmonic loads, working along the pipeline section's length. Such a section, between two successive supports, can be modelled as a simply supported beam along which a harmonic uniform distributed disturbing load $f(t) = f^0 \cos\omega t$ is working. Using the Lagrange equations, the following problems are approached in the present paper: the differential equation of motion which describes the pipeline section's vibration, the natural frequency formula and its values` dependence on various dimensional factors, the dynamic amplitude at the beam's middle in the forced vibration and, in the particular case when an axial force working at one beam's end occurs, the study of the manner in which this force can influence the natural frequencies` values. Such an axial force can be met when the temperature variations of the environment appear.

## 2. EQUATION OF MOTION

We consider a pipeline section having the loads as they are shown in figure 1.a. One knows that, in general, the differential equations of motion of the elastic systems with distributed mass, having the dynamic form deformation as a function of one parameter, can be determined using the Lagrange equations. We assume that the studied pipeline section is an elastic system with continuous distributed mass, having known geometrical characteristics (moment of inertia $I$, cross-sectional area $A$, mass per unit length $\overline{m}=m/L$) and having, at first sight, an infinite degrees of freedom. Assuming a dynamic form deformation depending on one parameter- the time, then the pipeline section can be solved as a single degree of freedom system. So, at a given distance $x$ (see fig. 1.c), in the case of the transverse vibrations, the dynamic deflection depends on time only ([5]) and its formula is given by:

$$\eta(x,t) = v(x) \cdot \eta(t) \tag{1}$$

where *v(x)* is the deflection in the point at the distance x, having a value equal to the unit at the middle of the beam (in this studied case). Knowing the equation of *v(x)*, the system can be transformed in a single degree of freedom system, with the motion described only by η(t).
Starting from the formula of the deflection v(x), known as it is established by the strength of materials methods ([6]), in the case of an uniform distributed load *q* (as it is shown in figure 1.b):

$$v(x) = \frac{ql^4}{24EI} \cdot \frac{x}{l} \cdot \left[1 - 2 \cdot \left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right] \tag{2}$$

the maximum deflection occurs at the half length of the pipeline section and it is given by:

$$v_{\max} = \frac{5ql^4}{384EI} \tag{3}$$

For a deflection equal to a unit of length, the intensity of the distributed load that produces this deflection is:

$$q = \frac{384EI}{5l^4} \qquad (4).$$

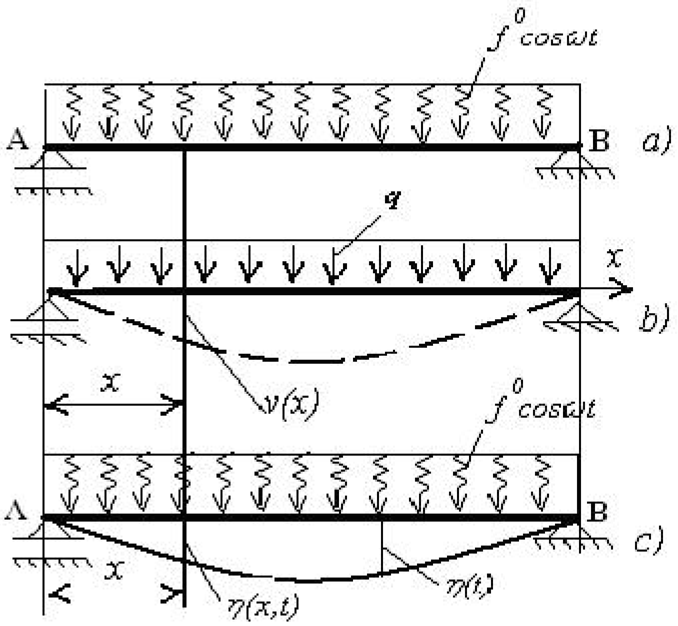


***Fig. 1.***

Using (4), the deflection (2) is given by the formula (5), which corresponds to that intensity of the load q that produces a maximum deflection equal to 1.

$$v(x) = \frac{16}{5} \cdot \frac{x}{l} \cdot \left[1 - 2\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right] \qquad (5).$$

In the case of the parameter of motion η(t), the Lagrange's equation of motion has the known formula:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\eta}}\right) - \frac{\partial E}{\partial \eta} + \frac{\partial F^d}{\partial \dot{\eta}} + \frac{\partial U}{\partial \eta} = Q_\eta \qquad (6),$$

where *E* is the kinetic energy, *U* – the strain energy and $Q_\eta$ - the generalized force. The kinetic energy is given by:

$$E = \frac{1}{2}\int_0^l \overline{m}\left[\frac{\partial\eta(x,t)}{\partial t}\right]^2 dx = \frac{1}{2}\int_0^l \overline{m}[v(x)]^2 \cdot \dot{\eta}^2 dx \qquad (7),$$

and:

$$\frac{\partial E}{\partial\dot{\eta}} = \int_0^l \overline{m}[v(x)]^2 \dot{\eta} dx = m^* \cdot \dot{\eta} \qquad (8),$$

where $m^*$ is the generalized mass:

$$m^* = \int_0^l \overline{m}[v(x)]^2 dx \qquad (9).$$

In this particular case, the generalized mass is given by:

$$m^* = \int_0^l \overline{m} \cdot \left[\frac{16}{5}\left(\frac{x}{l} - 2\left(\frac{x}{l}\right)^3 + \left(\frac{x}{l}\right)^4\right)\right]^2 dx = \frac{3968}{7875}\overline{m} \cdot l \approx 0{,}5038\overline{m} \cdot l \qquad (10).$$

The strain energy is determined with:

$$U = \frac{1}{2}\int_0^l EI\left[v''(x)\right]^2 \cdot \eta^2 dx \qquad (11),$$

and

$$\frac{\partial U}{\partial\eta} = k^* \cdot \eta \qquad (12),$$

where $k^*$ is the generalized stiffness:

$$k^* = \int_0^l EI\left[v''(x)\right]^2 dx = \int_0^l EI\left[\frac{192}{5} \cdot \left(\frac{x^2}{l^4} - \frac{x}{l^3}\right)\right]^2 dx = \frac{12288}{250}\frac{EI}{l^3} \approx 49{,}152\frac{EI}{l^3} \qquad (13).$$

In the case of the uniform distributed disturbing harmonic load, the generalized force $Q_\eta$ is given by:

$$Q_\eta = \int_0^l f(x,t)dx \cdot \frac{\partial\eta(x,t)}{\partial\eta} = \int_0^l f(x,t) \cdot v(x)dx = F_G \qquad (14),$$

where $F_G$ is the real generalized force, which, in this case, is given by:

$$F_G = \int_0^l f(x,t) \cdot v(x)dx = \int_0^l f^0 \cos\omega t \cdot \frac{16}{5}\left[\frac{x}{l} - 2\left(\frac{x}{l}\right)^3 + \left(\frac{x}{l}\right)^4\right]dx = \frac{16}{25} f^0 \cdot l \cdot \cos\omega t \qquad (15).$$

In the absence of the dumping forces ($F^d$ =0), the equation (6) has the following final form:

$$0{,}5038\overline{m} \cdot l \cdot \ddot{\eta}(t) + 49{,}152\frac{EI}{l^3} \cdot \eta(t) = \frac{16}{25} f^0 \cdot l \cdot \cos\omega t \qquad (16).$$

Formula (16) represents the differential equation that describes the transverse vibration of a pipeline section, considered to be simply supported at both ends and submitted to uniform distributed disturbing harmonic loads resulted from the fluids` pressure variations. It's obvious that $\overline{m}$ represents the sum between the steel's mass per unit length and the fluid's mass per unit length.

## 3. THE PIPELINE SECTION`S NATURAL FREQUENCY

The equation (16) can be also written as:

$$\ddot{\eta}(t) + 97{,}5625 \frac{EI}{\overline{m} \cdot l^4} \cdot \eta(t) = \frac{16}{25} f^0 \cdot l \cdot \cos \omega t \quad (17),$$

and, in the case of the free vibrations, when the term from the right side of (17) is zero, the natural frequency's formula is:

$$p = \sqrt{\frac{k^*}{m^*}} = \sqrt{97{,}5625 \frac{EI}{\overline{m} \cdot l^4}} \cong 9{,}87737 \sqrt{\frac{EI}{\overline{m} \cdot l^4}} \quad (18).$$

Comparing (18) with the natural frequency's formula obtained when calculating the pipeline section as a continuous mass beam ([5], §1 and [7]):

$$p = \left(\frac{\pi}{l}\right)^2 \cdot \sqrt{\frac{EI}{\overline{m}}} \cong 9{,}8696 \sqrt{\frac{EI}{\overline{m} \cdot l^4}} \quad (19),$$

one observes that the obtained values are very close. In table 1 are presented the geometrical characteristics of five different types of pipe, having the same wall thickness of 17,5 *mm* (according to standards DIN EN 10216/2-10220 and ASTM A106/ASME B36 10M), and the mass per unit length of the fluid corresponding to the inner cross sectional area of each pipe. Taking into account that the pipeline is full of fluid (gasoline, $\rho_b = 760 Kg/m^3$), in the formula (19), $\overline{m}$ is the sum between the steel's mass per unit length and the fluid's mass per unit length. In figure 2, the variation of the natural frequency of the pipeline section versus the length between the supports, for different nominal diameters of pipes with the same wall thickness, is presented. The utility of this graph appears when, knowing the pipeline section's length, one wants to determine the natural frequency in an easy manner. After that, the disturbing factor can be established so that important dynamic phenomena do not occur.

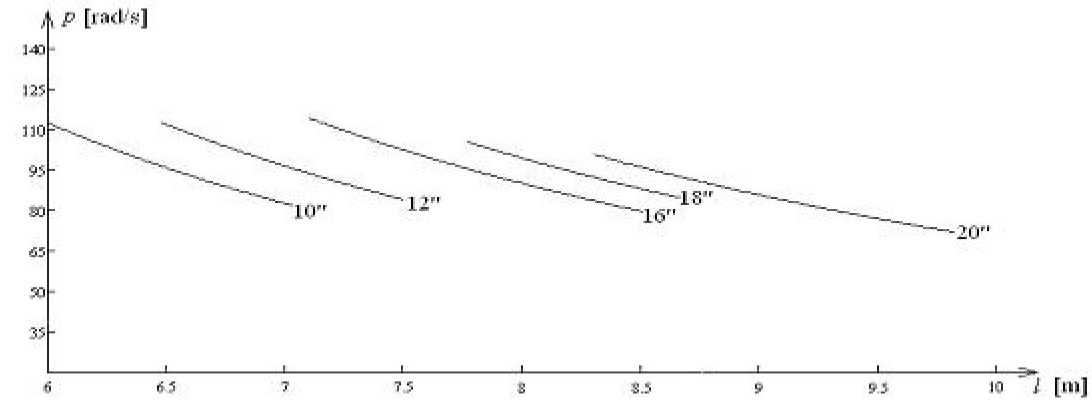


***Figure 2. Natural frequencies versus pipeline sections' length***

We ascertain that, function of the section's length, for the 10" pipe, the natural frequency has values between (82...112) *rad/s*, for 12" between (86...113) *rad/s*, for 16" between (84...114) *rad/s*, for 18" between (87...98) *rad/s* and for 20" between (75...101) *rad/s*. We observe the decrease of the natural frequencies` values together with the length's increase, finding the smallest values when the section has the maximum accepted length of the given pipeline section.

## 4. THE DYNAMIC AMPLITUDE IN THE FORCED VIBRATION

The dynamic amplitude in the forced vibration, in the absence of damping phenomena, is given by ([4]):

$$\eta_{din,\max} = \frac{F^0}{k^*} \cdot \psi \cdot \cos \omega t \qquad (20),$$

where ψ is the dynamic coefficient defined as:

$$\psi = 1/(1 - \frac{\omega^2}{p^2}) \qquad (21).$$

In this theoretical analysis, a ratio $\omega/p$= 0,75 will be taken into account. Using (13), (17) and (21) in the formula (20), we obtain the following final form for the dynamic amplitude:

$$\eta_{din,\max} = 0{,}0297 \frac{f^0 l^4}{EI} \qquad (22).$$

In figure 3, the variation of the dynamic amplitude (dynamic maximum deflection) for different types of pipes is presented. A maximum intensity $f^0$=3 *N/mm* for the disturbing distributed load and all the geometrical and physical data presented in table 1 (as the recommended lengths between two successive supports for every type of pipe) were taken into account.

***Table 1.***

| $D_n$ [inch] | $I$ [$mm^4$] | $\overline{m}_{pipe}$, [$kg/m$] | $\overline{m}_{fluid}$ [$kg/m$] | $L$ [$m$] | $A_{int}$, [$mm^2$] |
|---|---|---|---|---|---|
| 10” | $115{,}16\cdot10^6$ | 110,27 | 10,67 | 6...7 | $14{,}046\cdot10^3$ |
| 12” | $198{,}33\cdot10^6$ | 132,23 | 12,80 | 6,5...7,5 | $16{,}845\cdot10^3$ |
| 16” | $405{,}03\cdot10^6$ | 167,84 | 16,25 | 7,2...8,5 | $21{,}38\cdot10^3$ |
| 18” | $584{,}34\cdot10^6$ | 189,68 | 18,36 | 7,8...8,6 | $24{,}162\cdot10^3$ |
| 20” | $812{,}02\cdot10^6$ | 211,69 | 20,49 | 8,3...9,8 | $26{,}966\cdot10^3$ |

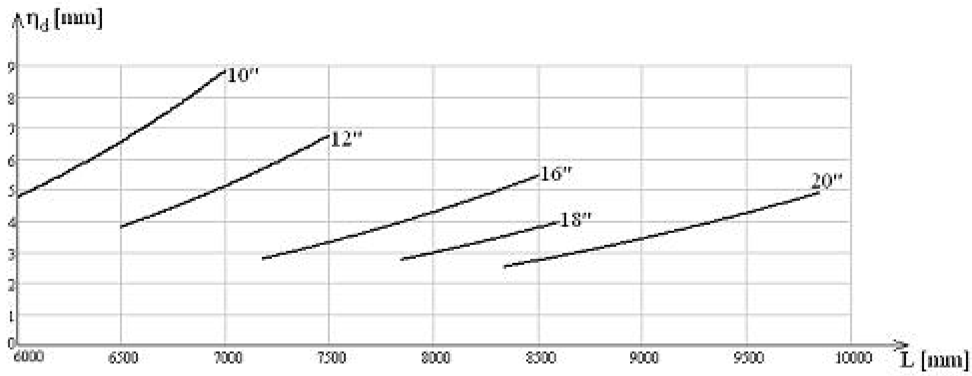


***Figure 3. The dynamic amplitude versus pipeline section’s length***

One can observe the amplitude’s increase together with the pipeline section’s length increase, for every type of pipe. The biggest interval of values can be observed for the 10” pipe, for which the variation is (4,8...8,9) *mm*, and the smallest for the 20” pipe, for which the interval of values is (2,7...4,9) mm. For the small diameter pipes, these amplitude’s values are big.

Depending on the disturbing distributed load’s intensity, every pipeline section’s dynamic amplitude can be easily determined, provided that the pipe’s natural frequency and the disturbing factor’s frequency are known (i.e. the ratio $\omega/p$ is known).

## 5. THE INFLUENCE OF THE AXIAL FORCE UPON THE NATURAL FREQUENCY

In the case of the existence of an axial (compression) force upon the pipeline section (fig. 4), considered to be applied at one section’s end and caused by external factors as the temperature's variations, the energy determined by this force and it's variations are given by([4], [5]):

$$U_F = -\frac{F}{2}\int_0^l \left[\frac{\partial \eta(x,t)}{\partial x}\right]^2 dx = -\frac{F}{2}\int_0^l [v'(x)]^2 dx \cdot \eta^2 \qquad (23),$$

$$\frac{\partial U_F}{\partial \eta} = -k_g^* \cdot \eta \qquad (24),$$

where $k_g^*$ is the generalized geometric stiffness given by:

$$k_g^* = F\int_0^l [v'(x)]^2 dx \qquad (25).$$

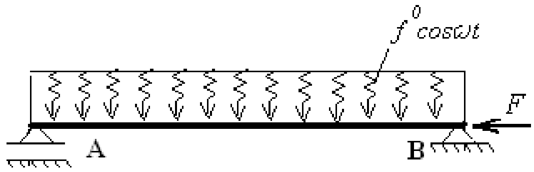


***Fig. 4. Model of the pipeline's section submitted to axial compressive force at one end***

In this studied case, the generalized geometric stiffness is:

$$k_g^* = F\int_0^l [v'(x)]^2 dx = F\int_0^l \left[\frac{16}{5}\left(\frac{1}{l} - \frac{6x^2}{l^3} + \frac{4x^3}{l^4}\right)\right]^2 dx = \frac{4352}{875}\frac{F}{l} \approx 4{,}9737\frac{F}{l} \qquad (26).$$

Expanding the equation of motion (16) for this case and using the combined generalized stiffness $k_C^*$, the differential equation, characteristic for the pipeline section's vibration when this one is submitted to uniform disturbing harmonic loads and compressive force at an end, is:

$$0{,}5038\overline{m} \cdot l \cdot \ddot{\eta}(t) + k_C^* \cdot \eta(t) = \frac{16}{25} f^0 \cdot l \cdot \cos \omega t \qquad (27).$$

The corresponding natural frequency is:

$$p^2 = \frac{\int_0^l EI[v''(x)]^2 dx - F\int_0^l [v'(x)]^2 dx}{\int_0^l \overline{m} \cdot [v'(x)]^2 dx} = \frac{k_C^*}{0{,}5038\overline{m} \cdot l} \qquad (28),$$

and it is in accordance with the energy method for determining the frequencies ([4], [7]).
The combined generalized stiffness is given by:

$$k_C^* = k^* - k_g^* = \frac{12288}{250}\frac{EI}{l^3} - \frac{4352}{875}\frac{F}{l} \qquad (29).$$

The condition *p=0* in (28) leads to the finding of the critical buckling force with (30). So, in these given conditions, the natural frequency of the pipeline sector will be obtained with (31).
In figure 5, the variation of the 12" pipeline's natural frequency, given by (31), is presented. The variation of the compressive force is assumed to be in the interval [0,1...0,4]$F_{cr}$, and the section's length is 7,5*m*.

$$F_{cr} = \frac{\int_0^l EI[v''(x)]^2 dx}{\int_0^l [v'(x)]^2 dx} = \frac{168}{17}\frac{EI}{l^2} \approx 9{,}882\frac{EI}{l^2} \tag{30}$$

$$p = \sqrt{\frac{k_C^*}{m^*}} = \sqrt{\frac{49{,}152EI - 4{,}9737Fl^2}{0{,}5038\overline{m} \cdot l^4}} \tag{31}$$

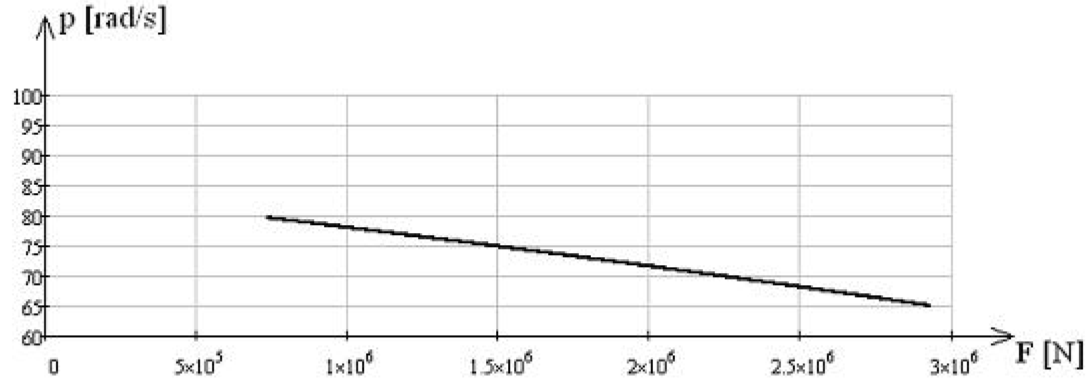


***Figure 5. Natural frequency versus axial compressive force (12", l= 7,5m)***

It can be observed that, together with the compressive force's increase from 0,1$F_{cr}$ to 0,4$F_{cr}$, the natural frequency decreases from 80 *rad/s* to 65 *rad/s*, noting that the natural frequency's value, in the absence of the compressive force, is 90 *rad/s*. It is obvious that, based on the formula (31), the variation of the natural frequency's values can be determined, for every type of pipeline and for every compressive force's intensity.

## 6. CONCLUSIONS

Using the Lagrange equations, in the present paper were determined: the differential equation of motion that describes the pipeline section's vibration; the natural frequency and its dependence on different pipeline's dimensional factors; the dynamic amplitude at the middle of the pipeline's section in the forced vibration; and, in the case of the existence of a compressive force at one sector's end, how this one can influence the natural frequencies` values. Such an axial compressive force may occur because of the environmental temperature's variations.

The formulas established in this paper are useful when calculating the dynamic parameters (natural frequency, dynamic deflection, stress intensity etc) for every type of pipeline charged with fluid, submitted to vibrations caused by harmonic uniform distributed disturbing loads and, when appropriate, to the influence of a compressive force applied at one sector's end.

Based on standards and [8], the natural frequencies` values, in dependence of the sector's length between two supports, were determined. The fluid considered to be circulated inside the pipeline was gasoline. We ascertained that, function of the section's length, for the 10" pipe, the natural frequency has values between (82...112) *rad/s*, for 12" between (86...113) *rad/s*, for 16" between (84...114) *rad/s*, for 18" between (87...98) *rad/s* and for 20" between (75...101) *rad/s*. We observed the decrease of the natural frequencies` values together with the length's increase, finding the smallest values when the section has the maximum accepted length of the given pipeline section.

For a value of 3 *N/mm* of the disturbing load, the dynamic deflection's variation depending on the pipeline section's length was analyzed. One can observe the amplitude's increase together with the pipeline section's length increase, for every type of pipe. The biggest interval of values can be observed for the 10" pipe, for which the variation is (4,8...8,9) *mm*, and the smallest for the 20" pipe, for which the interval of values is (2,7...4,9) *mm*. For the small diameter pipes, these amplitude's values are big.

After the analysis of the axial compressive force's influence upon the 12" pipeline's natural frequency, we observed that, together with the compressive force's increase from $0{,}1F_{cr}$ to $0{,}4F_{cr}$, the natural frequency decreased from 80 rad/s to 65 rad/s, noting that the natural frequency's value, in the absence of the compressive force, was 90 *rad/s*.

## BIBLIOGRAPHY

[1] Jong-Dar Yau - *Vibration Of Simply Supported Compound Beams To Moving Loads*, Journal of Marine Science and Technology, 2004, Vol. 12, No. 4, pp. 319-328.

[2] Lee H.P. - *Dynamic stability of a beam on multiple supports subject to pulsating conservative axial loads,* Engineering Computations, 1995 , Volume 12, Issue 4 , pp. 385 – 393.

[3] Murphy J. F. - *Transverse Vibration of a Simply Supported Beam with Symmetric Overhang of Arbitrary Length*, Journal of Testing and Evaluation, JTEVA, Vol. 25, No. 5, September 1997, pp. 522-524.

[4] Posea N. - *Calculul dinamic al structurilor*, Editura Tehnică, Bucureşti, 1991

[5] Posea N., Anghel Al., Popa I. – *Probleme de dinamica structurilor*. Editura didactică şi pedagogică R.A., Bucureşti, 1994.

[6] Posea N. - *Rezistenţa materialelor*, Editura Didactică şi Pedagogică, Bucureşti, 1979.

[7] Shabana A. A. - *Vibration of Discrete and Continuous Systems*. Second Edition, Springer-Verlag New York, Inc., 1997.

[8] * * * - *Codul tehnic al sectorului gazelor naturale*, ANRGN, Monitorul Oficial nr. 173 bis/27 februarie 2004.